

DC MOTOR CONTROL USING CHOPPER

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS OF THE DEGREE OF

**Bachelor of Technology
In
Electrical Engineering
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**Under the guidance
Of
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NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

CERTIFICATE

This is to certify that the progress report of the thesis entitled, “**CONTROL OF DC MOTOR USING CHOPPER**” submitted by **Shri Amir Faizy** in partial fulfillment of the requirements for the award of Bachelor of Technology degree in Electrical Engineering at the **National Institute of Technology Rourkela, India**, is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any degree or diploma.

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ABSTRACT

The speed of separately excited DC motor can be controlled from below and up to rated speed using chopper as a converter. The chopper firing circuit receives signal from controller and then chopper gives variable voltage to the armature of the motor for achieving desired speed. There are two control loops, one for controlling current and another for speed. The controller used is Proportional-Integral type which removes the delay and provides fast control. Modeling of separately excited DC motor is done. The complete layout of DC drive mechanism is obtained. The designing of current and speed controller is carried out. The optimization of speed controller is done using modulus hugging approach, in order to get stable and fast control of DC motor. After obtaining the complete model of DC drive system, the model is simulated using MATLAB(SIMULINK).The simulation of DC motor drive is done and analyzed under varying speed and varying load torque conditions like rated speed and load torque, half the rated load torque and speed, step speed and load torque and stair case load torque and speed.

Chapter 1

INTRODUCTION

Development of high performance motor drives are very essential for industrial applications. A high performance motor drive system must have good dynamic speed command tracking and load regulating response. DC motors provide excellent control of speed for acceleration and deceleration. The power supply of a DC motor connects directly to the field of the motor which allows for precise voltage control, and is necessary for speed and torque control applications.

DC drives, because of their simplicity, ease of application, reliability and favorable cost have long been a backbone of industrial applications. DC drives are less complex as compared to AC drives system. DC drives are normally less expensive for low horsepower ratings. DC motors have a long tradition of being used as adjustable speed machines and a wide range of options have evolved for this purpose. Cooling blowers and inlet air flanges provide cooling air for a wide speed range at constant torque. DC regenerative drives are available for applications requiring continuous regeneration for overhauling loads. AC drives with this capability would be more complex and expensive. Properly applied brush and maintenance of commutator is minimal. DC motors are capable of providing starting and accelerating torques in excess of 400% of rated ^[3].

D.C motors have long been the primary means of electric traction. They are also used for mobile equipment such as golf carts, quarry and mining applications. DC motors are conveniently portable and well fit to special applications, like industrial equipments and machineries that are not easily run from remote power sources ^[25].

D.C motor is considered a SISO (Single Input and Single Output) system having torque/speed characteristics compatible with most mechanical loads. This makes a D.C motor controllable over a wide range of speeds by proper adjustment of the terminal voltage. Now days, Induction motors, Brushless D.C motors and Synchronous motors have gained widespread use in electric traction system. Even then, there is a persistent effort towards making them behave like dc motors through innovative design and control techniques. Hence dc motors are always a good option for advanced control algorithm because the theory of dc motor speed control is extendable to other types of motors as well [3].

Speed control techniques in separately excited dc motor:

- By varying the armature voltage for below rated speed.
- By varying field flux should to achieve speed above the rated speed.

Different methods for speed control of DC motor:

- Traditionally armature voltage using Rheostatic method for low power dc motors.
- Use of conventional PID controllers.
- Neural Network Controllers.
- Constant power motor field weakening controller based on load-adaptive multi-input multi- output linearization technique (for high speed regimes).
- Single phase uniform PWM ac-dc buck-boost converter with only one switching device used for armature voltage control.
- Using NARMA-L2 (Non-linear Auto-regressive Moving Average) controller for the constant torque region.

Large experiences have been gained in designing trajectory controllers based on self-tuning and PI control. The PI based speed control has many advantages like fast control, low cost and simplified structure. This thesis mainly deals with controlling DC motor speed using Chopper as power converter and PI as speed and current controller.

Chapter 2

CHOPPER

2.1. DC CHOPPER

A chopper is a static power electronic device that converts fixed dc input voltage to a variable dc output voltage. A Chopper may be considered as dc equivalent of an ac transformer since they behave in an identical manner. As chopper involves one stage conversion, these are more efficient ^[2].

Choppers are now being used all over the world for rapid transit systems. These are also used in trolley cars, marine hoist, forklift trucks and mine haulers. The future electric automobiles are likely to use choppers for their speed control and braking. Chopper systems offer smooth control, high efficiency, faster response and regeneration facility ^[2].

The power semiconductor devices used for a chopper circuit can be force commutated thyristor, power BJT, MOSFET and IGBT. GTO based chopper are also used. These devices are generally represented by a switch. When the switch is off, no current can flow. Current flows through the load when switch is “on”. The power semiconductor devices have on-state voltage drop of 0.5V to 2.5V across them. For the sake of simplicity, this voltage drop across these devices is generally neglected ^[2].

As mentioned above, a chopper is dc equivalent to an ac transformer, have continuously variable turn's ratio. Like a transformer, a chopper can be used to step down or step up the fixed dc input voltage ^[2].

2.2. PRINCIPLE OF CHOPPER OPERATION

A chopper is a high speed “on” or “off” semiconductor switch. It connects source to load and load and disconnect the load from source at a fast speed. In this manner, a chopped load voltage as shown in Fig. is obtained from a constant dc supply of magnitude V_s . For the sake of highlighting the principle of chopper operation, the circuitry used for controlling the on, off periods is not shown. During the period T_{on} , chopper is on and load voltage is equal to source voltage V_s . During the period T_{off} , chopper is off, load voltage is zero. In this manner, a chopped dc voltage is produced at the load terminals ^[2].

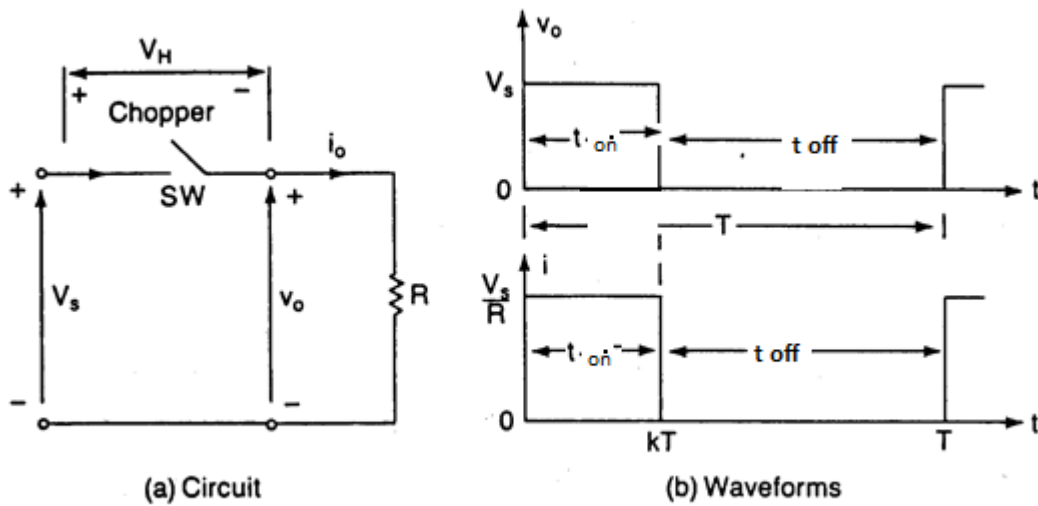


Figure1.Chopper Circuit and Voltage and Current Waveform.

Average Voltage, $V_o = (T_{on} / (T_{on} + T_{off})) * V_s$

$$= (T_{on} / T) * V_s$$

$$= \alpha V_s$$

T_{on} =on-time.

T_{off} =off-time.

$T = T_{on} + T_{off}$ = Chopping period.

$$\alpha = T_{on} / T_{off}.$$

Thus the voltage can be controlled by varying duty cycle α .

$$V_o = f * T_{on} * V_s$$

$f = 1/T = \text{chopping frequency}$.

2.3. CONTROL STRATEGIES ^[2]

The average value of output voltage V_o can be controlled through duty cycle by opening and closing the semiconductor switch periodically. The various control strategies for varying duty cycle are as following:

1. Time ratio Control (TRC) and
2. Current-Limit Control.

These are now explained below.

2.3.1. Time ratio Control (TRC)

In this control scheme, time ratio T_{on}/T (duty ratio) is varied. This is realized by two different ways called Constant Frequency System and Variable Frequency System as described below:

1. CONSTANT FREQUENCY SYSTEM ^[2]

In this scheme, on-time is varied but chopping frequency f is kept constant. Variation of T_{on} means adjustment of pulse width, as such this scheme is also called pulse-width-modulation scheme.

2. VARIABLE FREQUENCY SYSTEM ^[2]

In this technique, the chopping frequency f is varied and either (i) on-time T_{on} is kept constant or (ii) off-time T_{off} is kept constant. This method of controlling duty ratio is also called Frequency-modulation scheme.

2.3.2. CURRENT- LIMIT CONTROL ^[2]

In this control strategy, the on and off of chopper circuit is decided by the previous set value of load current. The two set values are maximum load current and minimum load current.

When the load current reaches the upper limit, chopper is switched off. When the load current falls below lower limit, the chopper is switched on. Switching frequency of chopper can be controlled by setting maximum and minimum level of current.

Current limit control involves feedback loop, the trigger circuit for the chopper is therefore more complex. PWM technique is the commonly chosen control strategy for the power control in chopper circuit.

2.4. GATE TURN OFF THYRISTOR AS A SWITCHING DEVICE ^{[2] [25]}

A GTO (Gate Turn Off) is a more versatile power-semiconductor device. It is like a Conventional Thyristor but with some added features . A GTO can easily be turned off by a negative gate pulse of appropriate amplitude. Thus, a GTO is a pn-pn device that can be turned on by a positive gate current and turned off by a negative gate current at the gate cathode terminals. Self –turn off capability of GTO makes it the suitable device for inverter and chopper applications.

2.4.1. Device Description: Normal thyristors are not fully controlled switches.

Thyristors can only be turned ON and but cannot be turned OFF. Thyristors are switched ON by a gate signal, but even after the gate signal is removed, the thyristor remains in the ON-state until any turn-off condition occurs, which can be the application of a reverse voltage to the terminals, or when the forward

Current flowing through goes below a certain threshold value known as the "Holding current". A thyristor behaves like a normal semiconductor diode after it is turned on.

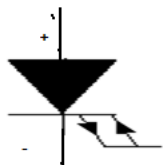


Figure2.Circuit Symbol of GTO ^[25].

The GTO can be turned-on by a gate signal, and can be turned-off by a gate signal of negative polarity. Turn on is accomplished by a positive current pulse between the gate and the cathode terminals. As the gate-cathode behaves like PN junction there will be some relatively small voltage drop between the terminals. The turn on process in GTO is however, not as reliable as an SCR and small positive gate current must be maintained even after turn on to improve reliability.

Turn off is achieved by a negative voltage pulse between the gate and cathode terminals. Some of the forward current (approx one-third to one-fifth) is stolen and used to induce a cathode-gate voltage which in turn induces the forward current to fall and the GTO switch off.

GTO thyristors suffer from long switch off times, whereby after the forward current falls, there is a long tail time where residual current continues to flow until all remaining charge from the device is taken away. This restricts the maximum switching frequency to approx 1 kHz. It should be noted that the turn off time of a comparable SCR is ten times that of a GTO. Thus switching frequency of GTO is much higher than SCR.

2.4.2. Comparison between GTO and Thyristor ^[2]:

A GTO has the following disadvantages as compared to a conventional thyristor:

- (i) Magnitude of Latching current and holding currents is more in a GTO.
- (ii) On state voltage drop and associated loss is more in a GTO.
- (iii) Gate drive circuit losses are more.
- (iv) Its reverse-voltage blocking capacity is less than its forward-voltage blocking capability. But this is no disadvantage to chopper circuit.

In spite of all these demerits, GTO has the following advantages over an SCR:

- (i) GTO has faster switching speed.
- (ii) Its surge current capability is comparable with an SCR.
- (iii) It has more di/dt rating at turn-on.
- (iv) GTO has lower size and weight as compare to SCR.
- (v) GTO unit has higher efficiency because an increase in gate drive power loss and on state loss is more than compensated by the elimination of forced commutation.
- (vi) GTO has reduced acoustical and electromagnetic noise due to elimination of commutation chokes.

Chapter 3

SEPARATELY EXCITED DC MOTOR

3.1. Basics of Separately Excited DC Motor ^[13]:

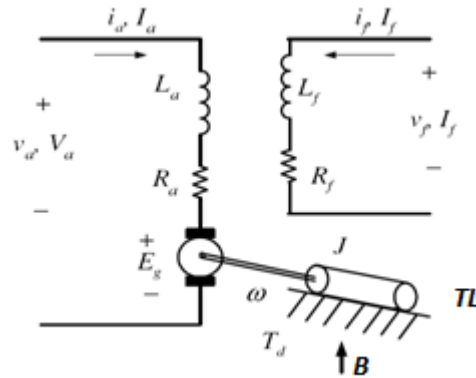


Figure3. Separately Excited DC motor ^[13].

- Separately Excited DC motor has field and armature winding with separate supply.
- The field windings of the dc motor are used to excite the field flux.
- Current in armature circuit is supplied to the rotor via brush and commutator segment for the mechanical work.
- The rotor torque is produced by interaction of field flux and armature current.

3.2. Operation of Separately excited DC motor ^[13]:

- When a separately excited dc motor is excited by a field current of i_f and an armature current of i_a flows in the circuit, the motor develops a back EMF and a torque to balance the load torque at a particular speed.
- The field current i_f is independent of the armature current i_a . Each winding is supplied separately. Any change in the armature current has no effect on the field current.

- The i_f is generally much less than the i_a .

3.3. FIELD AND ARMATURE EQUATIONS ^[13]:

Instantaneous field current:

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

where R_f and L_f are the field resistor and inductor, respectively

Instantaneous armature current :

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_g$$

where R_a and L_a are the armature resistor and inductance respectively

The motor back emf, which is also known as speed voltage, is expressed as :

$$e_g = K_v \omega i_f .$$

K_v is the motor voltage constant (in V/A - rad/Sec.
and ω is the motor speed (in rad/sec).

3.4. BASIC TORQUE EQUATION ^[13]:

The torque developed by the motor is :

$$T_d = K_t i_f i_a$$

where ($K_t = K_v$) is the torque constant.
(in V/A - rad/s)

Sometimes it is written as :

$$T_d = K_t \phi i_a$$

For normal operation, the developed torque must be equal to the load torque plus the friction and inertia, i.e.:

$$T_d = J \frac{d\omega}{dt} + B\omega + T_L$$

where

B : viscous friction constant, (N.m/rad/s)

T_L : load torque (N.m)

J : inertia of the motor (kg.m^2)

3.5. STEADY-STATE TORQUE AND SPEED ^[13]:

The motor speed can be easily derived:

$$\omega = \frac{V_a - I_a R_a}{K_v I_f}$$

If R_a is a small value (which is usual), or when the motor is lightly loaded, i.e. I_a is small,

$$\omega = \frac{V_a}{K_v I_f}$$

That is if the field current is kept constant, the motor speed depends only on the supply voltage.

The developed torque is:

$$T_d = K_t I_f I_a = B\omega + T_L$$

The required power is:

$$P_d = T_d \omega$$

3.6. TORQUE AND SPEED CONTROL ^[13]:

- From the above derivation important facts can be deduced for steady-state operation of DC motor.
- For a fixed field current, or flux (I_f) the torque demand can be satisfied by varying the armature current (I_a).
- The motor speed can be controlled by:
 - controlling V_a (voltage control)
 - controlling V_f (field control)
- These observation lead to the application of variable DC voltage for controlling the speed and torque of DC motor.

3.7. VARIABLE SPEED OPERATION ^[13]:

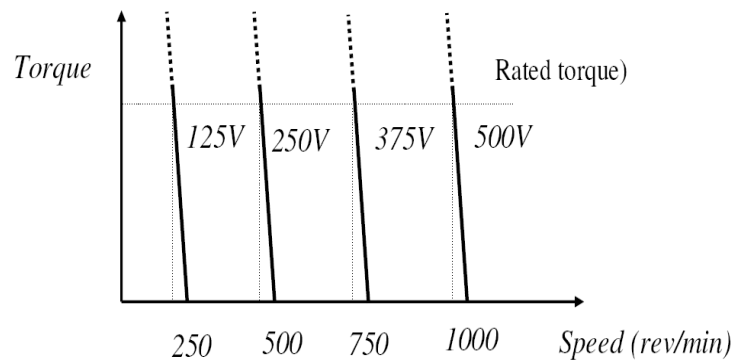


Figure 4: Torque Vs Speed Characteristic For Different Armature Voltages

- Family of steady state torque speed curves for a range of armature voltage can be drawn as above.

- The speed of DC motor can simply be set by applying the correct voltage.
- The speed variation from no load to full load (rated) can be quite small. It depends on the armature resistance.

3.8. BASE SPEED AND FIELD-WEAKENING^[13]:

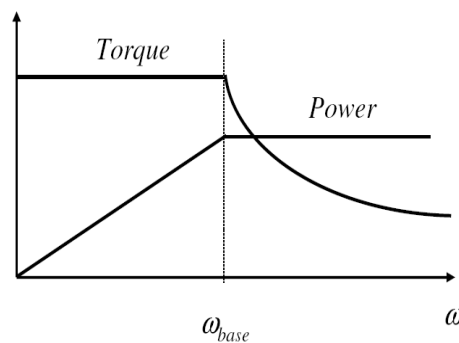


Figure 5: Torque Vs Speed And Power Vs Speed Characteristic Of Separately Excited DC Motor

- **Base speed:** (ω_{base})
 - The speed which correspond to the rated V_a , rated I_a and rated I_f .
- **Constant Torque** region ($\omega < \omega_{base}$)
 - I_a and I_f are maintained constant to met torque demand. V_a is varied to control the speed.

Power increases with speed.
- **Constant Power** region ($\omega > \omega_{base}$)
 - V_a is maintained at the rated value and I_f is reduced to increase speed. However, the power developed by the motor (= torque x speed) remains constant. This phenomenon is known as Field weakening.

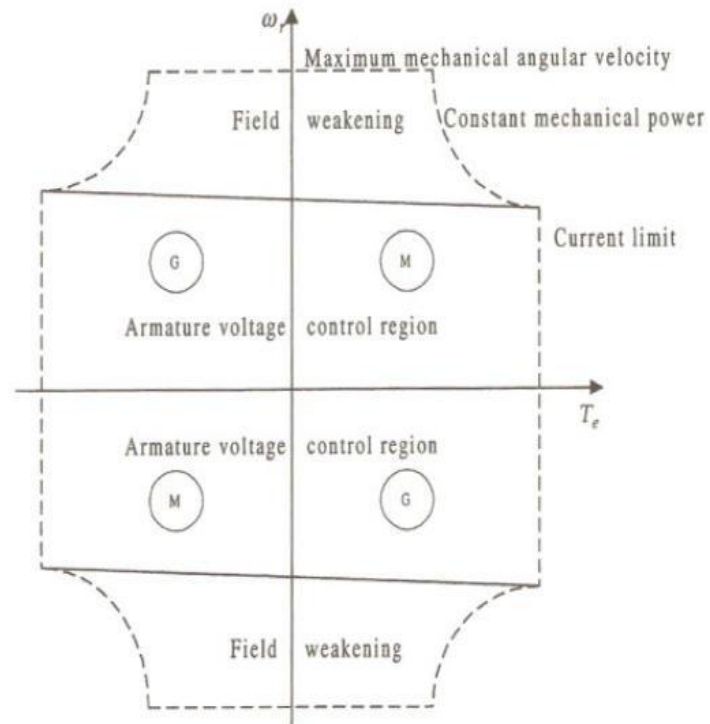


Figure 6: Typical Operating Regions Of Separately Excited DC Machines^[13]

Chapter 4

MODELING OF DC MOTOR FOR DRIVE SYSTEM

4.1. BASIC IDEA

The basic principle behind DC motor speed control is that the output speed of DC motor can be varied by controlling armature voltage for speed below and up to rated speed keeping field voltage constant. The output speed is compared with the reference speed and error signal is fed to speed controller. Controller output will vary whenever there is a difference in the reference speed and the speed feedback. The output of the speed controller is the control voltage E_c that controls the operation duty cycle of (here the converter used is a **Chopper**) converter. The converter output give the required V_a required to bring motor back to the desired speed. The Reference speed is provided through a potential divider because the voltage from potential divider is linearly related to the speed of the DC motor. The output speed of motor is measured by Tacho-generator and since Tacho voltage will not be perfectly dc and will have some ripple. So, we require a filter with a gain to bring Tacho output back to controller level ^[1].

The basic block diagram for DC motor speed control is show below:

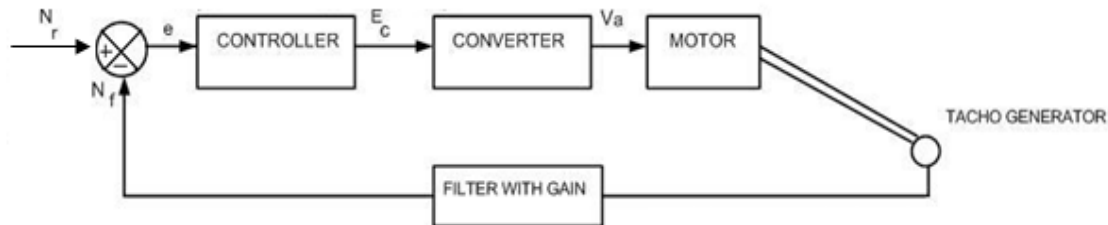


Figure7.Closed loop system model for speed control of dc motor ^[1].

4.2. MODELING OF SEPARATELY EXCITED DC MOTOR ^[1]

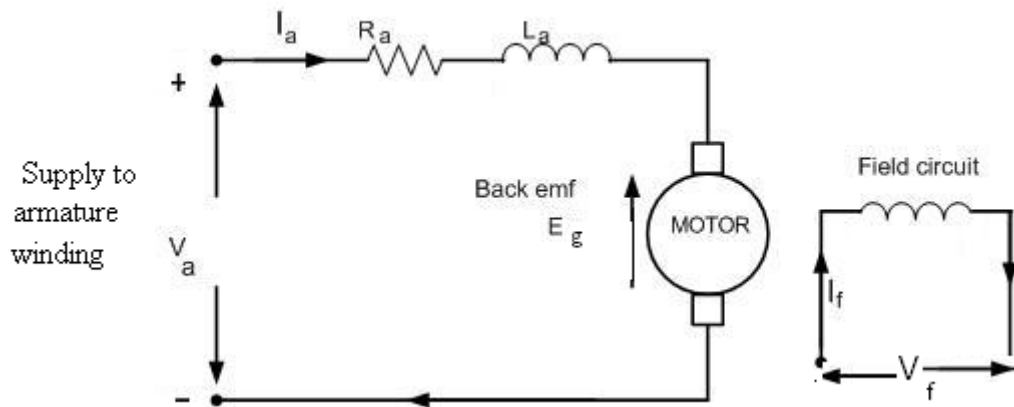


Figure8.Separately Excited DC motor model.

The armature equation is shown below:

$$V_a = E_g + I_a R_a + L_a (dI_a/dt)$$

The description for the notations used is given below:

1. V_a is the armature voltage in volts.
2. E_g is the motor back emf in volts.
3. I_a is the armature current in amperes.
4. R_a is the armature resistance in ohms.
5. L_a is the armature inductance in Henry.

Now the torque equation will be given by:

$$T_d = Jd\omega/dt + B\omega + T_L$$

Where:

1. T_L is load torque in Nm.
2. T_d is the torque developed in Nm.

3. J is moment of inertia in kg/m².

4. B is friction coefficient of the motor.

5. ω is angular velocity in rad/sec.

Assuming absence (negligible) of friction in rotor of motor, it will yield:

$$B=0$$

Therefore, new torque equation will be given by:

$$T_d = Jd\omega/dt + T_L \quad \text{----- (i)}$$

Taking field flux as Φ and (Back EMF Constant) K_v as K. Equation for back emf of motor will be:

$$E_g = K \Phi \omega \quad \text{----- (ii)}$$

$$\text{Also, } T_d = K \Phi I_a \quad \text{----- (iii)}$$

From motor's basic armature equation, after taking Laplace Transform on both sides, we will get:

$$I_a(S) = (V_a - E_g)/(R_a + L_a S)$$

Now, taking equation (ii) into consideration, we have:

$$\Rightarrow I_a(s) = (V_a - K\Phi\omega)/R_a(1 + L_a S/R_a)$$

$$\text{And, } \omega(s) = (T_d - T_L)/JS = (K\Phi I_a - T_L)/JS$$

Also, The armature time constant will be given by:

$$(\text{Armature Time Constant}) T_a = L_a/R_a$$

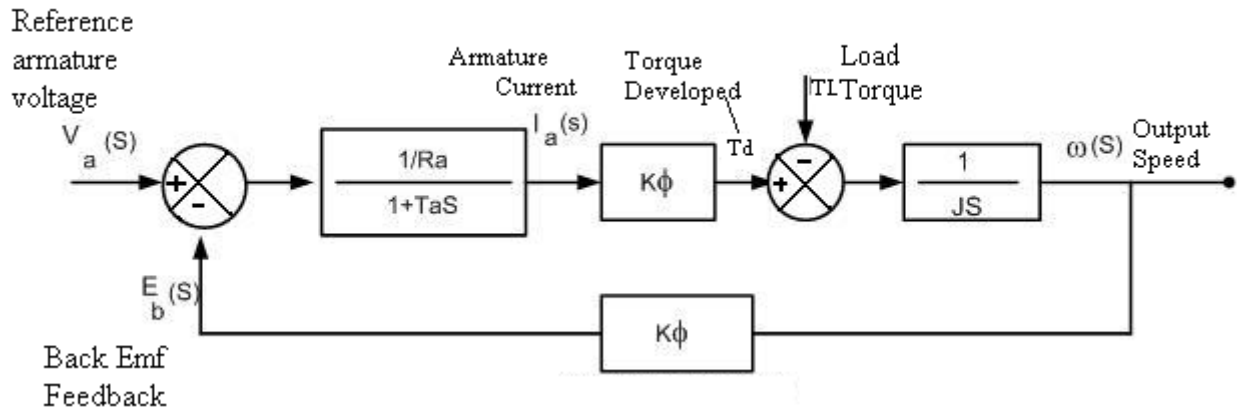


Figure 9. Block Model of Separately Excited DC Motor ^[1]

After simplifying the above motor model, the overall transfer function will be as given below:

$$\omega(s) / V_a(s) = [K\Phi / R_a] / JS(1+T_a S) / [1 + (K^2\Phi^2 / R_a) / JS(1+T_a S)]$$

Further simplifying the above transfer function will yield:

$$\omega(s) / V_a(s) = (1 / k\Phi) / \{ 1 + (k^2\Phi^2 / R_a) / JS(1+T_a S) \} \text{ ----- (iv)}$$

Assuming, $T_m = JR_a / (k\Phi)^2$ as electromechanical time constant ^[1].

Then the above transfer function can be written as below:

$$\omega(s) / V_a(s) = (1 / k\Phi) / [ST_m (1+ST_a) + 1] \text{ ----- (v)}$$

Let us assume that during starting of motor, load torque $T_L = 0$ and applying full voltage V_a

Also assuming negligible armature inductance, the basic armature equation can be written as:

$$V_a = K\Phi\omega(t) + I_a R_a$$

At the same time Torque equation will be:

$$T_d = Jd\omega/dt = K\Phi I_a \text{ ----- (vi)}$$

Putting the value of I_a in above armature equation:

$$V_a = K\Phi\omega(t) + (Jd\omega/dt)R_a / K\Phi$$

Dividing on both sides by $K\Phi$,

$$V_a/K\Phi = \omega(t) + JR_a(d\omega/dt)/(K\Phi)^2 \text{ -----(vii)}$$

$V_a/K\Phi$ is the value of motor speed under no load condition. Therefore,

$$\omega(\text{no load}) = \omega(t) + JR_a(d\omega/dt)/(K\Phi)^2 = \omega(t) + T_m(d\omega/dt)$$

$$\text{Where, } K\Phi = K_m(\text{say})$$

And,

$$T_m = JR_a/(K\Phi)^2 = JR_a/(K_m)^2$$

$$\text{Therefore, } J = T_m (K_m)^2 / R_a \text{ ----- (viii)}$$

From motor torque equation, we have:

$$\omega(s) = K_m I_a(s) / JS - \underline{T_L} / \underline{JS} \text{ ----- (ix)}$$

From equation (viii) and (ix), we have:

$$\omega(s) = [(R_a / K_m) I_a(s) - T_L R_a / (K_m)^2] (1/T_m(s))$$

Now, Replacing $K\Phi$ by K_m in equation (v), we will get:

$$\omega(s)/V_a(s) = (1/K_m) / (1 + ST_m + S^2 T_a T_m) \text{ ----- (x)}$$

Since, the armature time constant T_a is much less than the electromechanical time constant T_m ,
 $(T_a \ll T_m)$ ^[1]

Simplifying, $1 + ST_m + S^2T_aT_m \approx 1 + S(T_a + T_m) + S^2T_aT_m = (1 + ST_m)(1 + ST_a)$

The largest time constant will play main role in delaying the system when the transfer function is in time constant form. To compensate that delay due to largest time constant we can use PI controller as speed controller. It is because the zero of the PI controller can be chosen in such a way that this large delay can be cancelled. In Control system term a time delay generally corresponds to a lag and zero means a lead, so the PI controller will try to compensate the whole system ^[1].

Hence, the equation can be written as:

$$\omega(s)/V_a(s) = (1/K_m)/((1 + ST_m)(1 + ST_a)) \text{ -----(xi)}$$

T_m and T_a are the time constants of the above system transfer function which will determine the response of the system. Hence the dc motor can be replaced by the transfer function obtained in equation (xi) in the DC drive model shown earlier.

Chapter 5

CONTROLLER DESIGN

5.1. CONTROLLER FUNDAMENTALS ^[15]:

The controller used in a closed loop provides a very easy and common technique of keeping motor speed at any desired set-point speed under changing load conditions. This controller can also be used to keep the speed at the set-point value when, the set-point is ramping up or down at a defined rate. The essential addition required for this condition to the previous system is a means for the present speed to be measured.

In this closed loop speed controller, a voltage signal obtained from a Tacho-generator attached to the rotor which is proportional to the motor speed is fed back to the input where signal is subtracted from the set-point speed to produce an error signal. This error signal is then fed to work out what the magnitude of controller output will be to make the motor run at the desired set-point speed. For example, if the error speed is negative, this means the motor is running slow so that the controller output should be increased and vice-versa ^[15].

5.2. DECIDING THE TYPE OF CONTROLLER ^[15]

The control action can be imagined at first sight as something simple like if the error speed is negative, then multiply it by some scale factor generally known as gain and set the output drive to the desired level. But this approach is only partially successful due to the following reason: if the motor is at the set-point speed under no load there is no error speed so the motor free runs. If a load is applied, the motor slows down and a positive error speed is observed. Then the output increases by a proportional amount to try and restore the desired speed. However, when the motor speed recovers, the error reduces drastically and so does the drive level. The result is that the motor speed will stabilize at a speed below the set-point speed at which the load is balanced by the product of error speed and the gain. This basic technique discussed above is known as "**proportional control**" and it has limited use as it can never force the motor to run exactly at the set-point speed ^[15].

From the above discussion an improvement is required for the correction to the output which will keep on adding or subtracting a small amount to the output until the motor reaches the set-point. This effect can be done by keeping a running total of the error speed observed for instant at regular interval (say 25ms) and multiplying this by another gain before adding the result to the proportional correction found earlier. This approach is basically based on what is effectively the integration of the error in speed.

Till now we have two mechanisms working simultaneously trying to correct the motor speed which constitutes a PI (proportional-integral) controller. The proportional term does the job of fast-acting correction which will produce a change in the output as quickly as the error arises. The integral action takes a finite time to act but has the capability to make the steady-state speed error zero.

A further refinement uses the rate of change of error speed to apply an additional correction to the output drive. This is known as Derivative approach. It can be used to give a very fast response to sudden changes in motor speed. In simple PID controllers it becomes difficult to generate a derivative term in the output that has any significant effect on motor speed. It can be deployed to reduce the rapid speed oscillation caused by high proportional gain. However, in many controllers, it is not used. The derivative action causes the noise (random error) in the main signal to be amplified and reflected in the controller output. Hence the most suitable controller for speed control is PI type controller ^[15].

5.3. Importance of Current Controller in a DC drives system ^[1]:

When the machine is made to run from zero speed to a high speed then motor has to go to specified speed. But due to electromechanical time constant motor will take some time to speed up. But the speed controller used for controlling speed acts very fast. Speed feedback is zero initially. So this will result in full controller output E_c and hence converter will give maximum

voltage. So a very large current flow at starting time because back Emf is zero at that time which sometime exceeds the motor maximum current limit and can damage the motor windings. Hence there is a need to control current in motor armature. To solve the above problem we can employ a current controller which will take care of motor rated current limit. The applied voltage V_a will now not dependent on the speed error only but also on the current error. We should ensure that V_a is applied in such a way that machine during positive and negative torque, does not draw more than the rated current. So, an inner current loop hence current controller is required.

5.4. Representation of Chopper in Transfer function form:

Since chopper takes a fixed DC input voltage and gives variable DC output voltage. It works on the principle Pulse Width Modulation technique ^[2]. There is no time delay in its operation. Hence, it can be represented by a simple constant gain K_t .

5.5. COMPLETE LAYOUT FOR DC MOTOR SPEED CONTROL ^{[1] [3]}

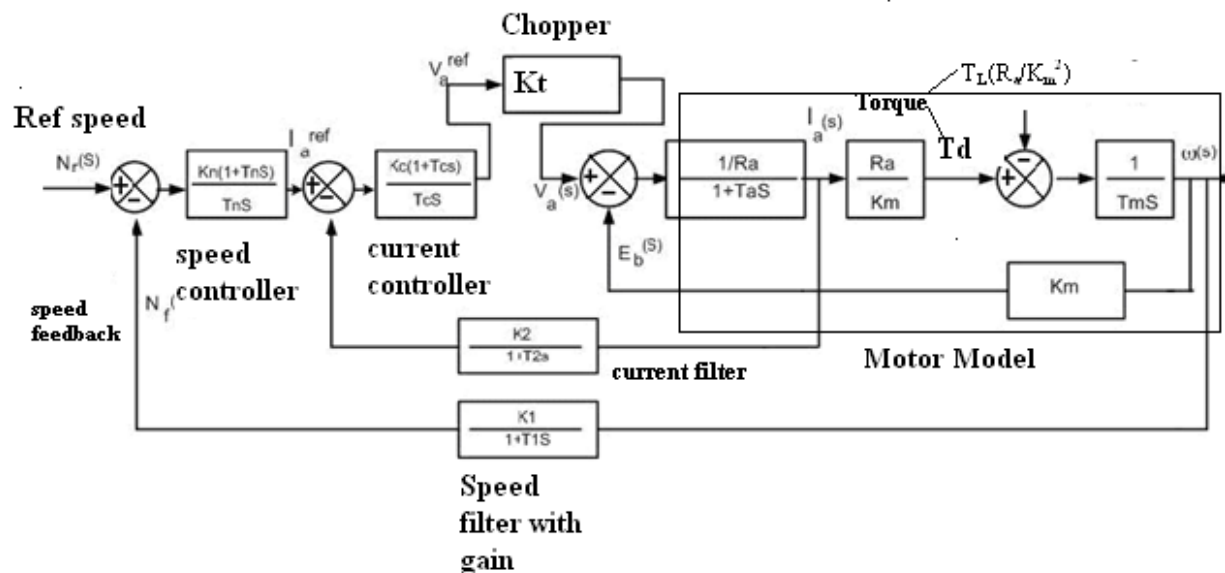


Figure 10. Complete layout for DC motor speed control ^{[1] [3]}.

5.6. CURRENT CONTROLLER DESIGN ^[1]:

We need to design current controller for the extreme condition when back emf is zero that is during starting period because at that time large current flows through the machine.

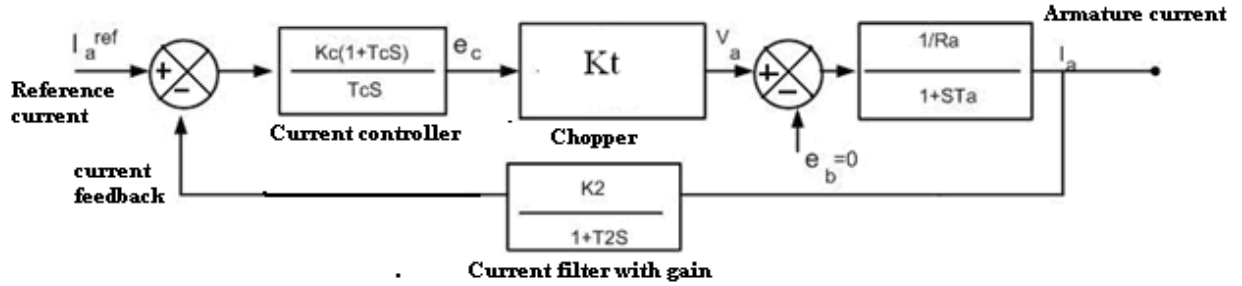


Figure 11. Block Model for Current Controller Design ^[1].

Transfer function of the above model:

$$I_a(s)(f)/I_a(s)ref = \{ [K_c(1+T_cS)/T_cS] (K_t) [(1/R_a)/(1+ST_a)] / \{ 1 + [K_c(1+T_cS)/T_cS] K_t [(1/R_a)/(1+ST_a)] [K_2/(1+T_2S)] \} \} \text{-----} \quad (xii)$$

Here, T_c (Current Controller Parameter) can be varied as when required. T_c should be chosen such that it cancels the largest time constant in the transfer function in order to reduce order of the system ^[1]. Now, the response will be much faster. So, let us assume

$$\boxed{T_c = T_a}$$

Now, putting this value in equation (xii)

$$I_a(S)(f)/I_a(S)(ref) = \{ K_c(K_t/T_a R_a)(1+T_2S) \} / \{ S(1+T_2S) + (K_c K_t K_2)/T_a R_a \} \text{---(xiii)}$$

Let, $K_o = (K_c K_t / T_a R_a)$

$$I_a(S)(f)/I_a(S)(ref) = K_o(1+T_2S) / [S^2 T_2 + S + K_o K_2] \text{----- (xiv)}$$

Where T_2 corresponds filter lag. Dividing T_2 on R.H.S:

$$I_a(S) (f)/ I_a(S) (ref) = \{(K_o/T_2) (1+T_2S)\} / [S^2+S/T_2 + K_oK_2/T_2] \quad \text{----- (xv)}$$

Characteristic Equation:-

$$S^2+(S/T_2)+(K_oK_2/T_2)\approx S^2+2\epsilon\omega+\omega^2$$

$$\text{here, } \omega = \sqrt{(K_oK_2)/T_2}$$

$$\epsilon = 1/(2T\omega) = 1/2\sqrt{(T_2K_2K_o)}$$

Since, it is a second order system. So, to get a proper response ϵ should be 0.707^{[4][6]}.

$$\text{So, } 1/\sqrt{2} = 1/2\sqrt{(T_2K_2K_o)} \Rightarrow K_o = 1/(2K_2T_2) = K_cK_t / (R_aT_a)$$

$$\boxed{K_c = (R_aT_a) / (2K_tK_2T_2)}$$

Here,

$$K_o = K_c K_t / (R_aT_a) = 1/(2K_2T_2) \Rightarrow K_oK_2 = 1/2T_2$$

$$\text{Now, from equation-(xiv): } I_a(S) (f)/ I_a(S) (ref) = \{(1/K_2) (1+T_2S)\} / [2S^2T_2^2+2ST_2 + 1] \quad \text{--- (xvi)}$$

We can see that the zero in the above equation may result in an overshoot. Therefore, we will use a time lag filter to cancel its effect. The current loop time constant is much higher than filter time constant. Hence a small delay will not affect much

$$\{I_a(S) (f)/ I_a(S) (ref)\} (1+ST_2) = \{(1/K_2) (1+T_2S)\} / [2S^2T_2^2+2ST_2 + 1]$$

Hence,

$$\boxed{I_a(S) (f) / I_a(S) (ref) = (1/K_2) / (2S^2T_2^2+2ST_2+1)}$$

5.7. SPEED CONTROLLER DESIGN ^[1]:

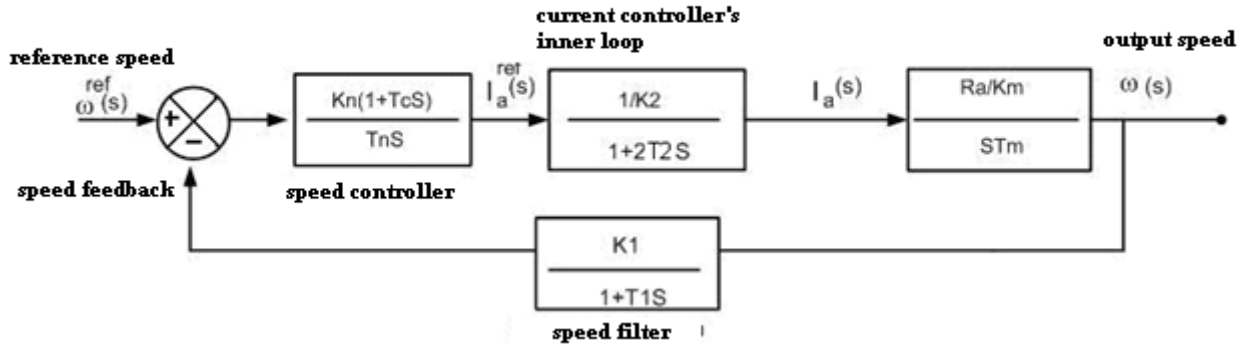


Figure 12. Block model for Speed Controller design ^[1].

Now, converting the block model in transfer function, we will get:

$$\omega(s)/\omega(s)(\text{ref.})=$$

$$(K_n/K_2)(R_a/K_m T_m T_n)(1+T_n S/(1+2T_2 S)S^2)/\{1+(K_n R_a/K_2 K_m T_m T_n)(1+T_n S/(1+2T_2 S)S^2)(K_1/(1+T_1 S))\} \quad \text{----- (xviii)}$$

Here, we have the option to T_n such that it cancels the largest time constant of the transfer function ^[1]. So,

$$T_n = 2T_2$$

Hence, equation --- (xviii) will be written as:

$$\omega(s)/\omega(s)(\text{ref.})=(K_n R_a/K_2 K_m T_m T_n)(1+T_1 S)/\{K_2 K_m T_n S^2(1+T_1 S)+K_n R_a K_1\}$$

$$\text{Ideally, } \omega(s) = 1/S (S^2 + \alpha s + \beta)$$

The damping constant is zero in above transfer function because of absence of S term, which results in oscillatory and unstable system. To optimize this we must get transfer function whose gain is close to unity ^{[1] [4] [6]}.

5.8. Modulus Hugging Approach for Optimization of Speed Controller

Transfer Function ^[1]:

If the variable to be controlled rapidly reaches the desired value then dynamic performance of the control system is considered as good. For any frequency variation within bandwidth of the input variable, the output should follow the input variable instantaneously for achieving unity gain.

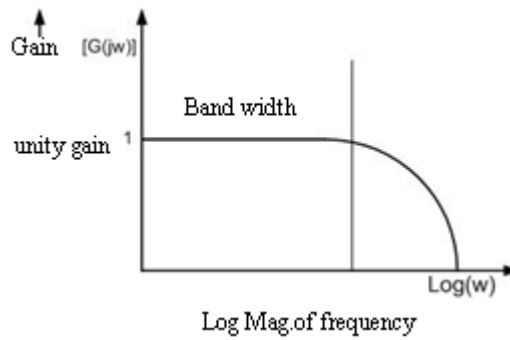


Figure 13. Gain Vs Frequency Waveform ^[1].

The process of making output close to input variable so as to obtain unity gain for wide frequency range is termed as **Modulus Hugging** ^[1].

Considering equation (xviii):

$$\omega(s)/\omega_s(\text{ref.}) = \{(K_n R_a)(1 + T_n S)(1 + T_1 S)\} / \{S^2 T_m T_n K_2 K_m (1 + 2T_2 S)(1 + T_1 S) + (K_n R_a K_1)(1 + T_n S)\}$$

$$\text{Here, } (1 + 2T_2 S)(1 + T_1 S) = 1 + T_1 S + 2T_2 S + 2T_2 T_1 S^2 \approx 1 + S(2T_2 + T_1) + 2T_2 T_1 S^2 \approx 1 + S(2T_2 + T_1)$$

Here, T_1 and T_2 are smaller time constants. So their product can be approximated to zero.

$$\text{So, } 1 + S(2T_2 + T_1) = 1 + \delta S. \quad \text{Assuming, } \delta = (2T_2 + T_1) \text{ and } K_o = (K_n R_a / K_2 K_m)$$

Then,

$$\omega(s)/\omega(s)(\text{ref.}) = \{(K_n R_a / K_2 K_m)(1 + T_n S)(1 + T_1 S)\} / \{S^3 T_m T_n \delta + S^2 T_m T_n + (K_o K_1 T_n)S + K_o K_1\}$$

The above transfer function is of third order. The terms $(1 + T_n S)$ and $(1 + T_1 S)$ in the denominator will be cancelled by using filters ^[1].

Taking a standard 3rd order system:

$$G(j\omega) = (b_0 + j\omega b_1) / [a_0 + j\omega a_1 + (j\omega)^2 a_2 + (j\omega)^3 a_3] \quad [1]$$

for low frequency $b_0 = a_0$ and $b_1 = a_1$

$$|G(j\omega)| = (a_0^2 + \omega^2 a_1^2) / (a_0^2 + \omega^2 (a_1^2 - 2a_0 a_2) + \omega^4 (a_2^2 - 2a_1 a_3) + \omega^6 (a_3^2))^{1/2}$$

Now, Modulus hugging principle, $|G(j\omega)| = 1$; for that coefficients of ω^2 and ω^4 are made equal to zero.

$$\text{So, } a_1^2 = 2a_0 a_2 \text{ \& } a_2^2 = 2a_1 a_3 \quad \text{----- (A)}$$

We need to use filters on the $\omega(s)$ (ref) side to cancel $(1 + T_n S)(1 + T_1 S)$ term:

$$\omega(s) / \{\omega(s)(\text{ref}) (1/(1 + T_n S))(1/(1 + T_1 S))\} = (K_n R_a / K_2 K_m)(1 + T_n S)(1 + T_1 S) / (S^3 T_m T_n \delta + S^2 T_m T_n + (K_o K_1 T_n)S + K_o K_1) \quad \text{----- (xix)}$$

Now, from optimization condition in - (A), we get:-

$$\Rightarrow (K_o K_1 T_n)^2 = 2 * K_o K_1 * T_m T_n$$

$$\Rightarrow K_o K_1 T_n = 2 T_m$$

$$\Rightarrow T_m = K_o K_1 T_n / 2 \quad \text{----- (xx)}$$

$$\text{Also, } (T_m T_n)^2 = 2 * T_m T_n \delta K_o K_1 T_n$$

$$\Rightarrow T_m = 2 \delta K_o K_1$$

$$\Rightarrow T_n K_o K_1 / 2 = 2 * \delta K_o K_1$$

$$\Rightarrow T_n = 4 \delta = (2 T_2 + T_1) \quad \text{----- (xxi)}$$

From equation (xx) and (xxi):

$$T_m = 2 K_o K_1 \delta = 2 (K_n R_a / K_2 K_m) K_1 \delta$$

$$K_n = T_m K_m K_2 / (2 K_1 R_a \delta) \quad \text{----- (xxii)}$$

Now, putting the values of K_n and K_m in the main transfer function, we get:

$$\omega(s)(f) / \{\omega(s)(\text{ref}) = 1 / (K_1 + 4 \delta K_1 + 8 s^2 \delta K_1 + 8 s^3 \delta K_1)$$

Chapter 6

PROBLEM STATEMENT

A separately excited DC motor with name plate ratings of 320KW, 440V (DC), 55 rad/sec is used in all simulations. Following parameter values are associated with it.

- Moment of Inertia, $J = 85 \text{ Kg-m}^2$.
- Back EMF Constant = 9 Volt-sec/rad.
- Rated Current = 715 A.
- Maximum Current Limit = 1000 A.
- Resistance of Armature, $R_a = 0.0241 \text{ ohm}$.
- Armature Inductance, $L_a = 0.718 \text{ mH}$.
- Speed Feedback Filter Time Constant ^[1], $T_1 = 25 \text{ ms}$.
- Current Filter Time Constant ^[1], $T_2 = 3.5 \text{ ms}$.

Current Controller Parameter ^[1]:

Current PI type controller is given by: $K_c \{(1 + T_c S)/T_c S\}$

Here, $T_c = T_a$ and $K_c = R_a T_a / (2K_2 K_t T_2)$

$$T_a = L_a / R_a = 0.718 \times 10^{-3} / 0.0241 = 29.79 \text{ ms}.$$

For analog circuit maximum controller output is $\pm 10 \text{ Volts}$ ^[1]. Therefore, $K_t = 440/10 = 44$.

Also, $K_2 = 10/1000 = 1/100$.

Now, putting value of R_a , T_a , K_2 , K_t and T_2 we get: $K_c = 0.233$.

Speed Controller Parameter ^[1]:

Speed PI type controller is given by: $K_n \{(1 + T_n S)/T_n S\}$

Here, $T_n = 4\delta = 4(T_1 + 2T_2) = 4(25 + 7) = 128 \text{ ms}$.

Also, $K_n = T_m K_m K_2 / (2K_1 R_a \delta)$.

$$K_1 = 10/55 = 0.181.$$

$$T_m = J R_a / K_m = 85 \times 0.0241 / 9 = 22.7 \text{ ms}.$$

$$\text{Now, } K_n = (22.7 \times 9 \times 1) / (2 \times 0.181 \times 0.0241 \times 32 \times 100) = 6.15$$

Chapter 7

MATLAB SIMULATION, RESULTS AND ANALYSIS

7.1.Simulink Model:

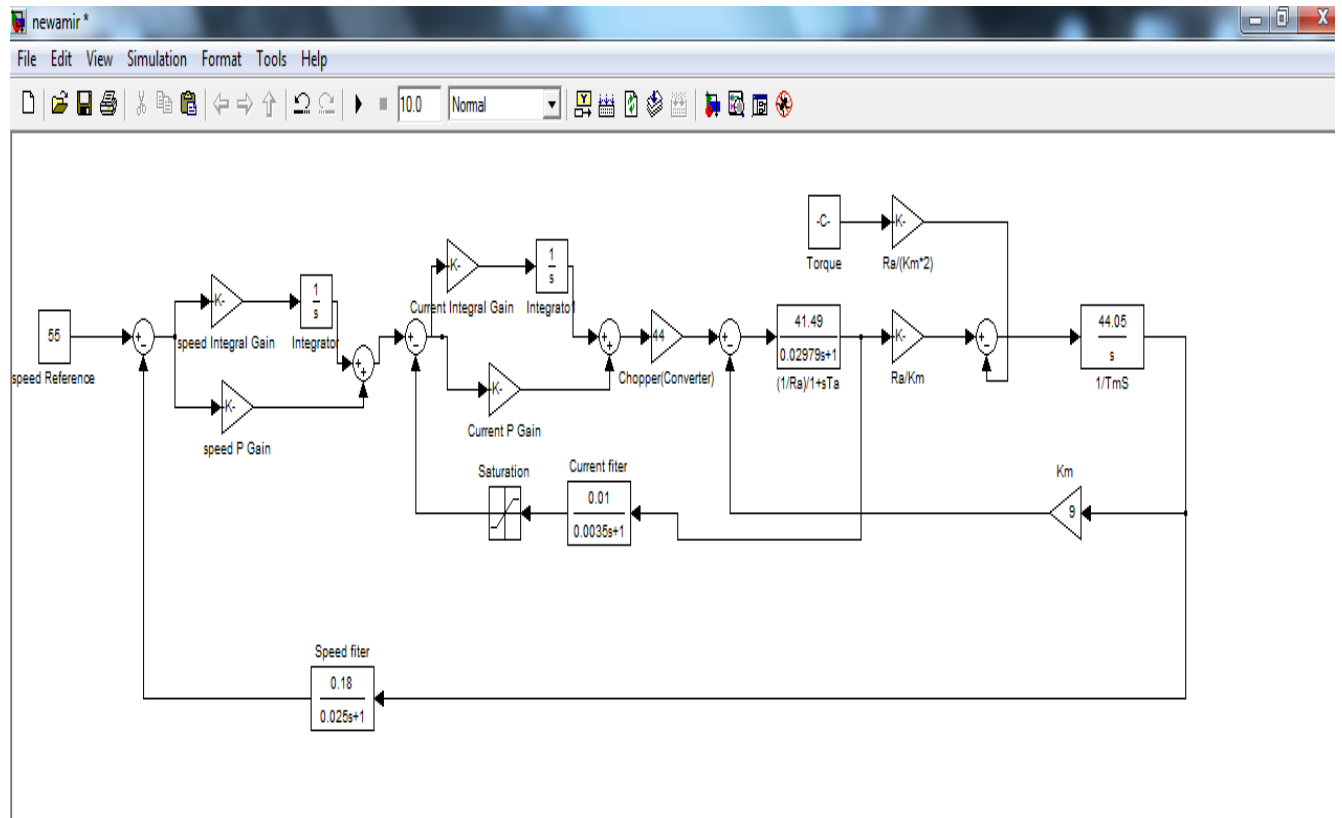


Fig14.Simulink Model for Speed Control of Separately Excited DC motor using Chopper Converter (without filter after reference speed)

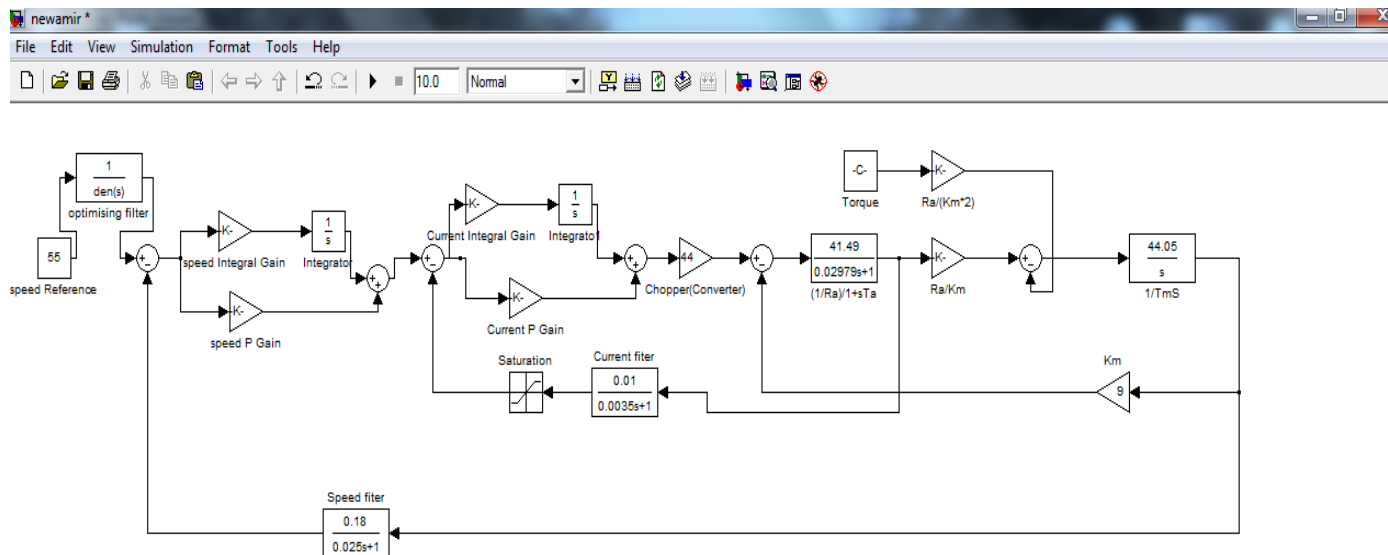
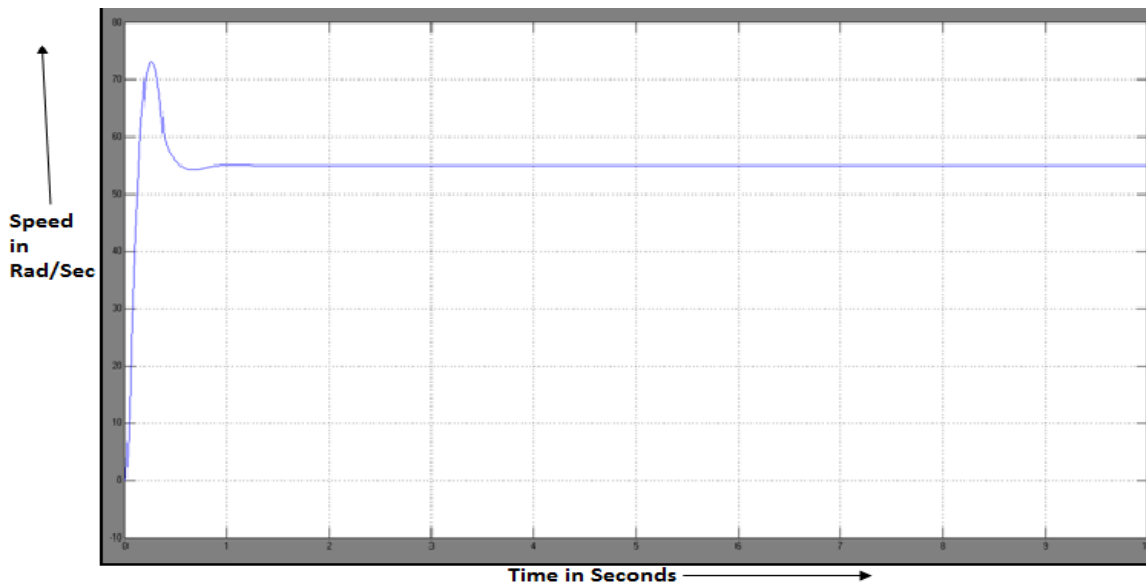
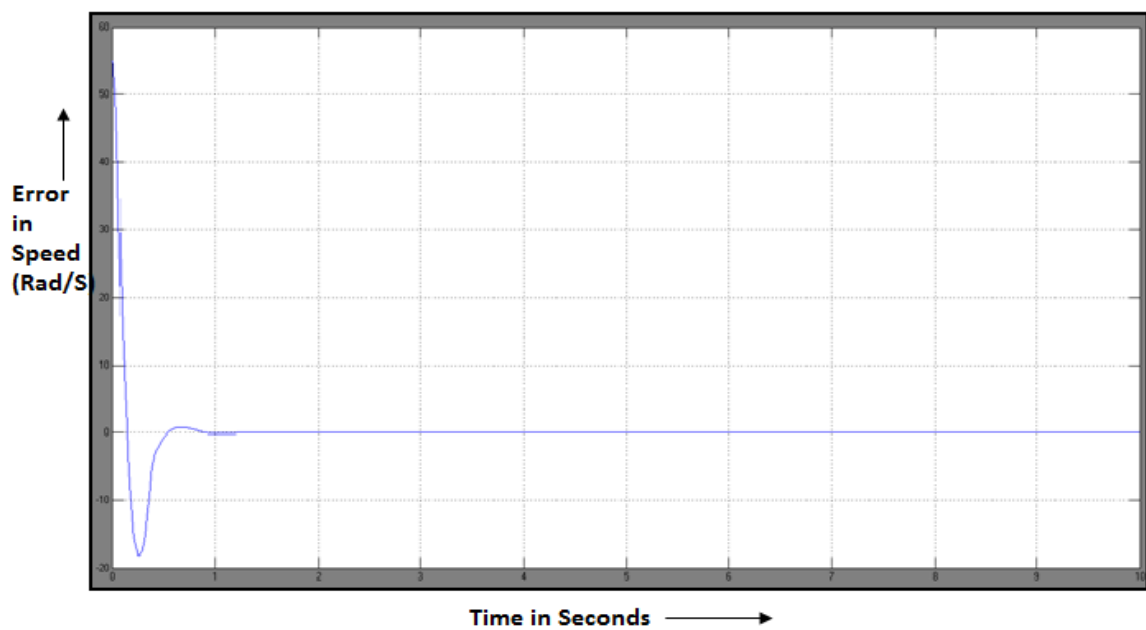


Fig15.Simulink Model for Speed Control of Separately Excited DC motor using Chopper Converter (with filter after reference speed)

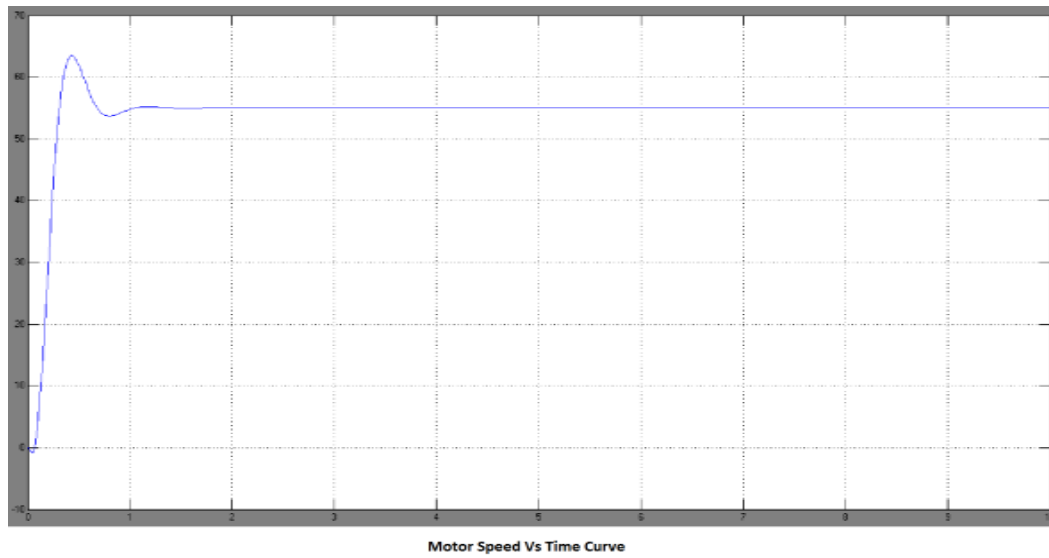
7.2. Graph1. Speed Response at reference speed same as rated speed and full Load (without Filter):



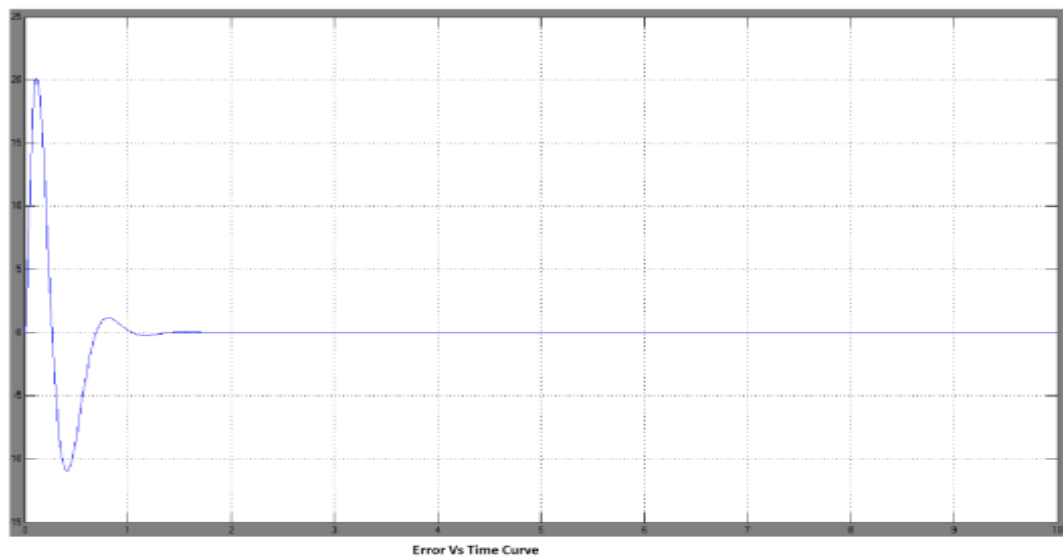
Graph2. Error in Speed Response at reference speed same as rated speed and full Load (without Filter):



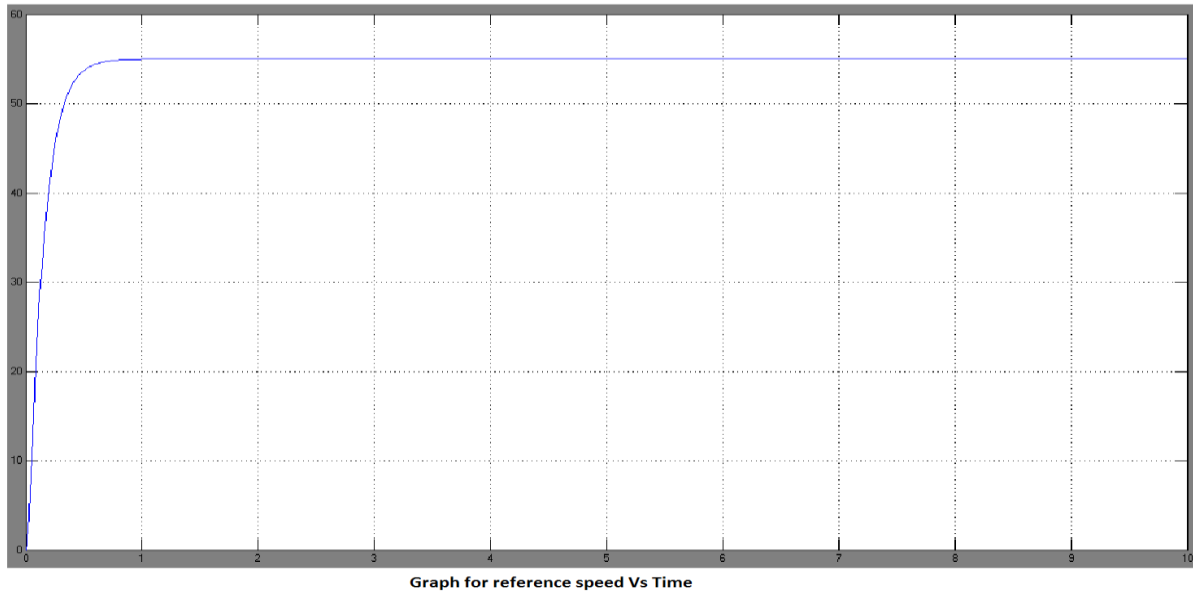
Graph3.Speed Response at reference speed same as rated speed and full Load (with Filter):



Graph4.Error in Speed Response at reference speed same as rated speed and full Load (with Filter):

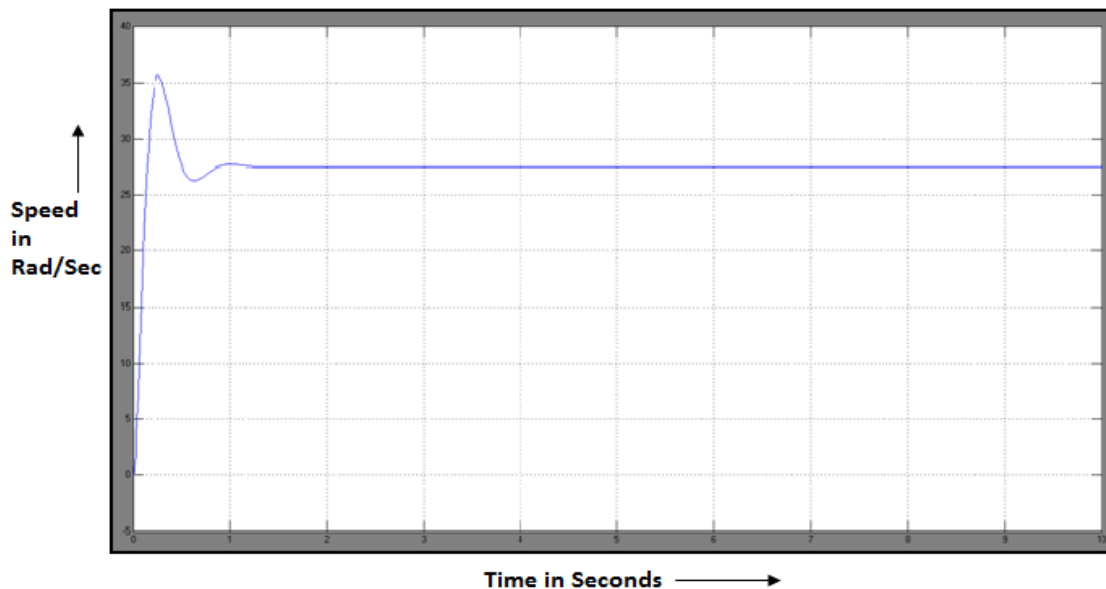


Graph5.Reference Speed Vs Time while using Filter after reference:

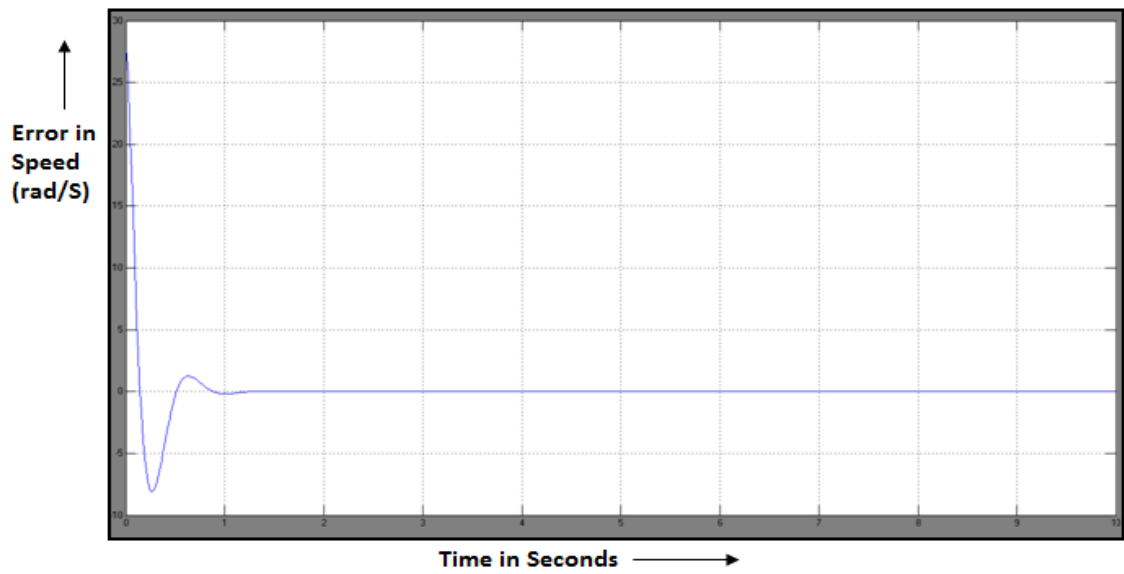


7.3. Analysis for Graph1to5: From above simulation results, it is clear that the SIMULINK model without filter (Graph 1&2) after reference speed gives larger overshoot in speed before settling to steady state and faster response than the model using filter (Graph 3&4) after reference speed. The error in later starts from zero because reference takes time to reach desired value (shown in graph5) due to introduction of filter.

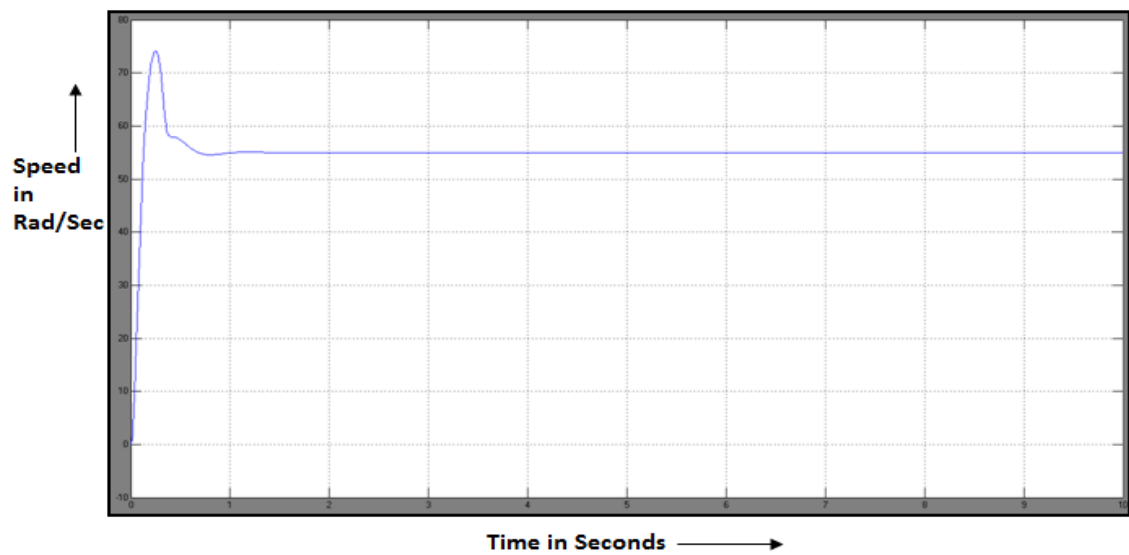
7.4. Graph6.Speed Response at reference speed half the rated speed and full Load:



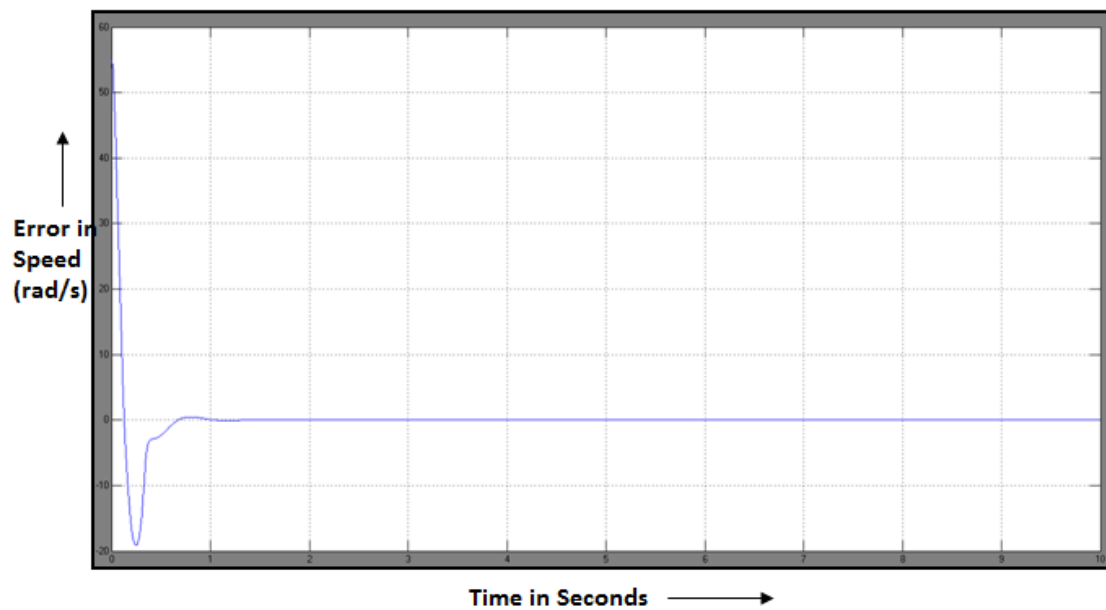
Graph7.Error in Speed Response at reference speed half the rated speed and full Load:



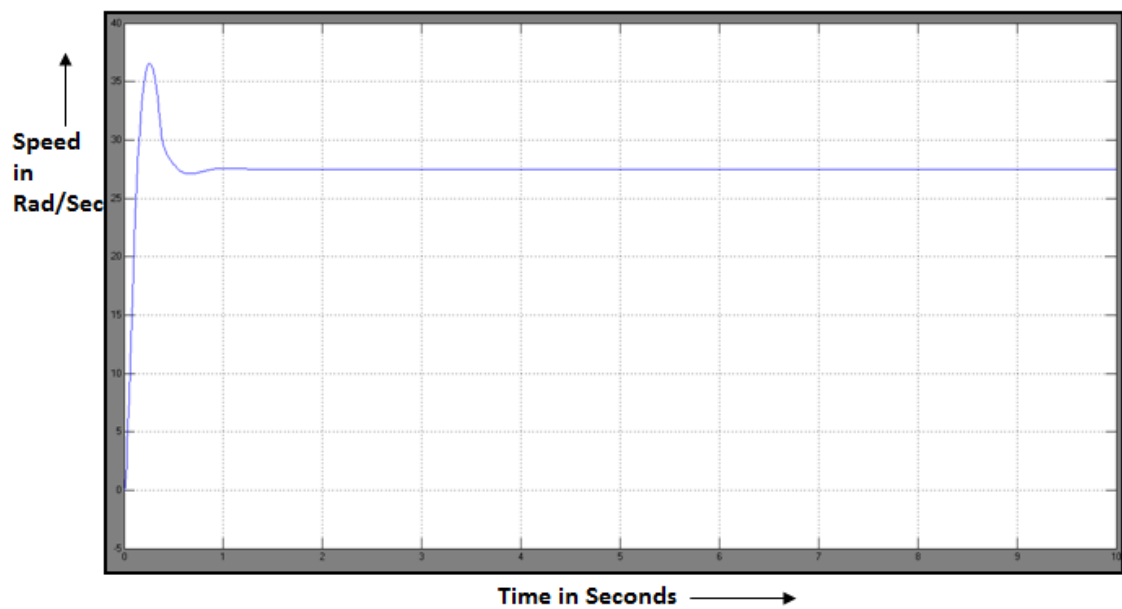
Graph8.Speed Response at reference speed same as rated speed and half of full Load :



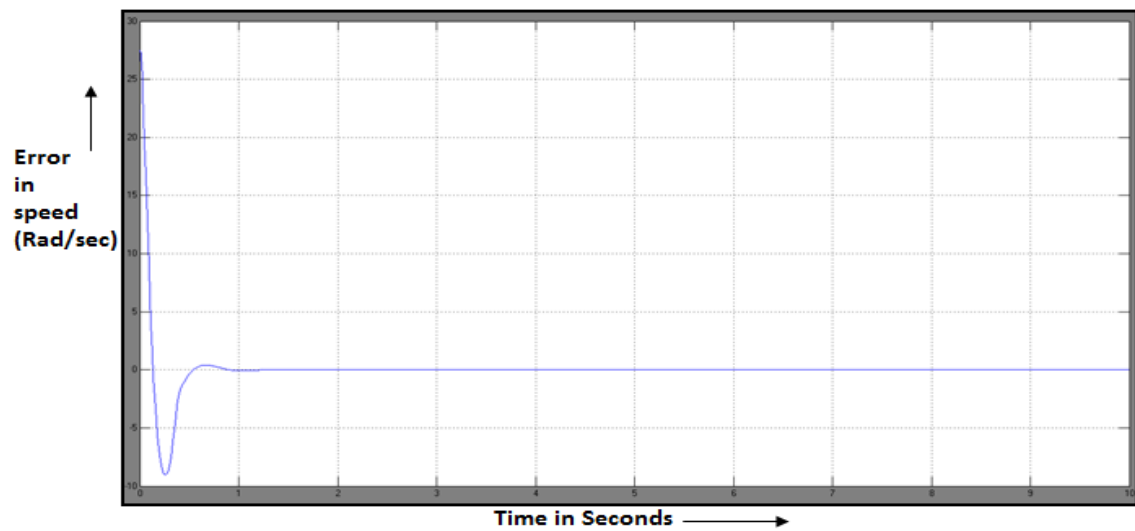
Graph9.Error in Speed Response at reference speed same as rated speed and half of full Load:



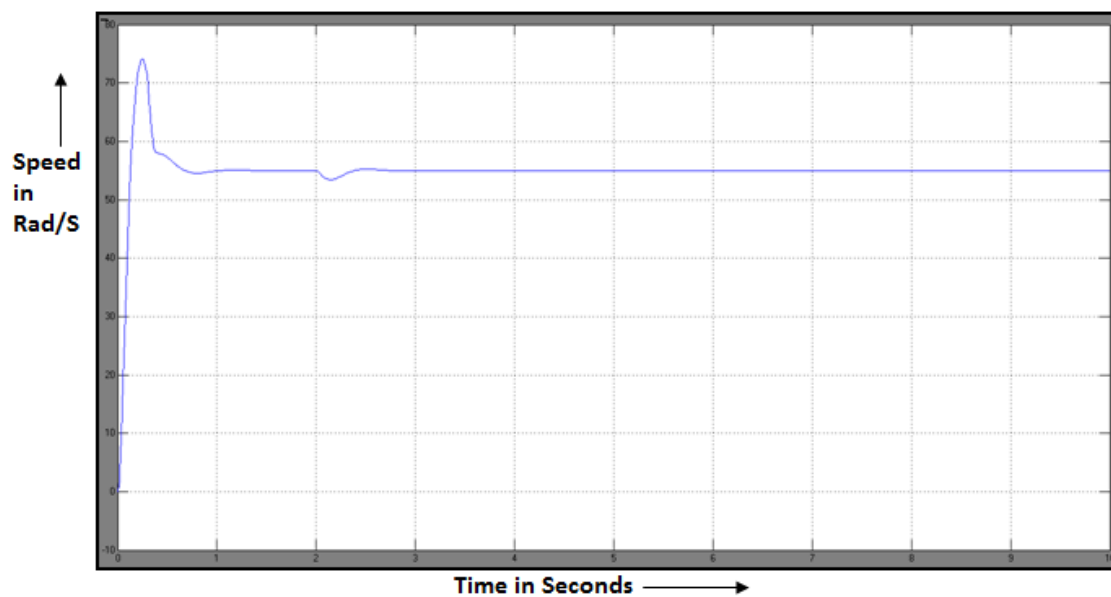
Graph10.Speed Response at reference speed of half the rated speed and half of full Load:



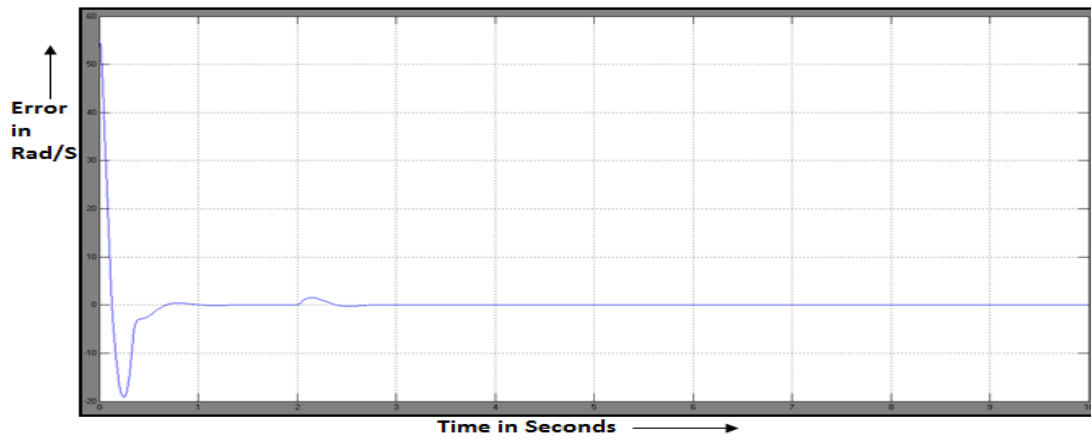
Graph11. Error in Speed Response at reference speed of half the rated speed and half of full Load:



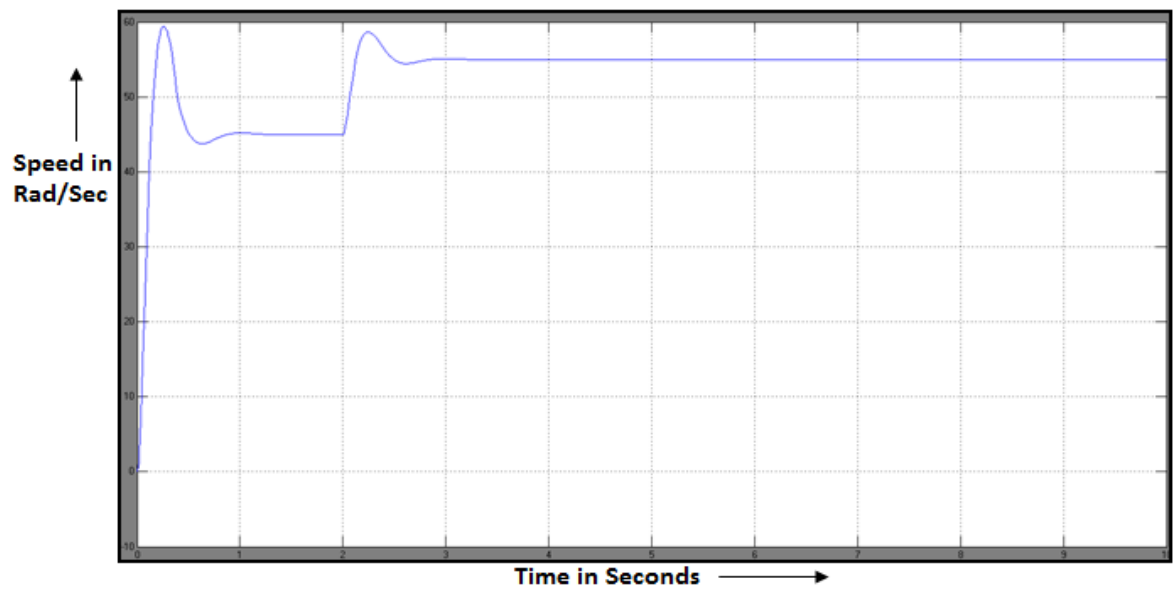
Graph12. Speed Response at reference speed same as rated speed and step torque Load:



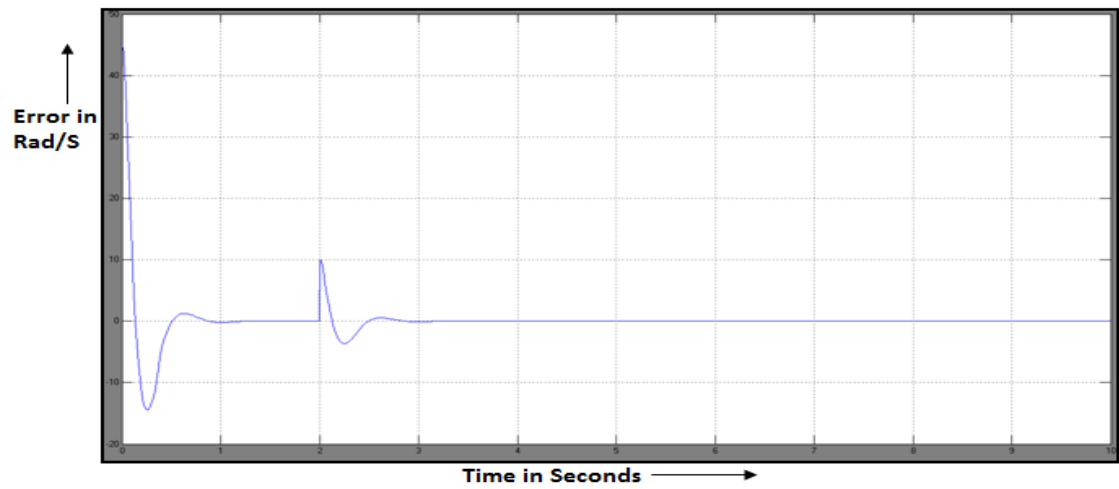
Graph13. Error in Speed Response at reference speed same as rated speed and step torque Load:



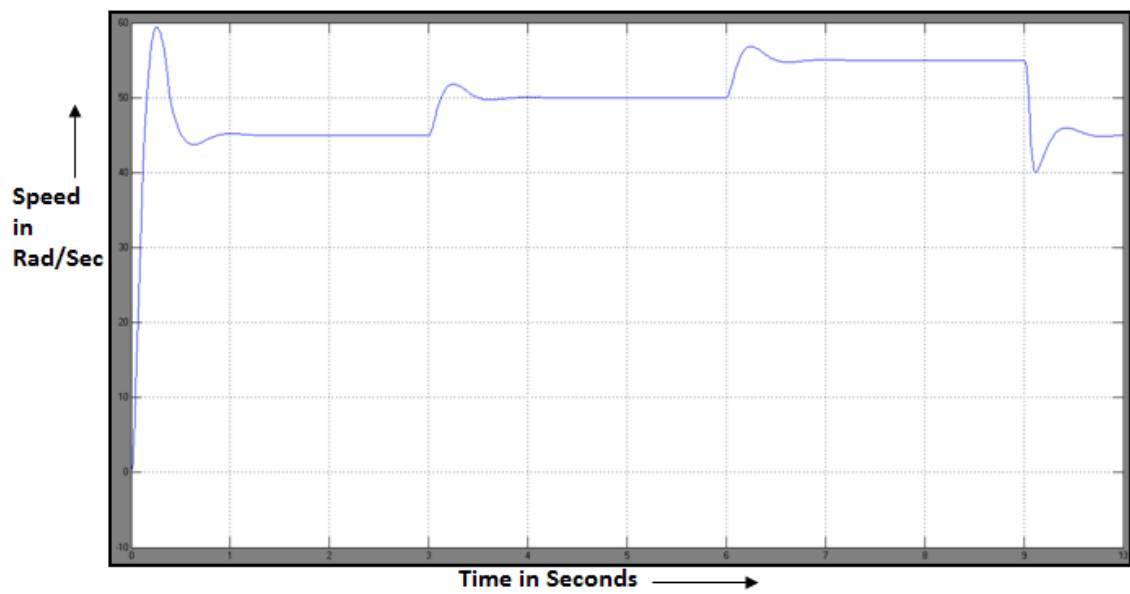
Graph14. Speed Response at step reference speed and constant torque Load:



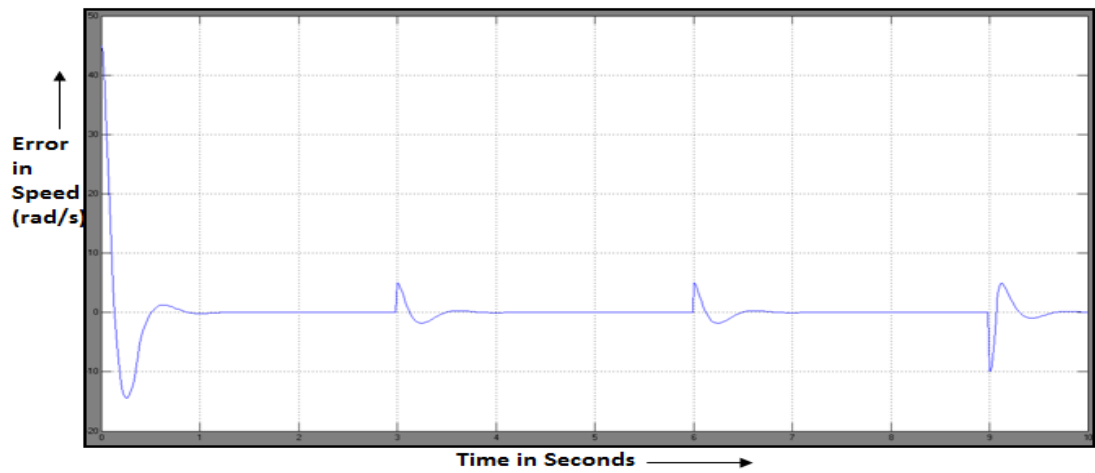
Graph15.Error in Speed Response at step reference speed and constant torque Load:



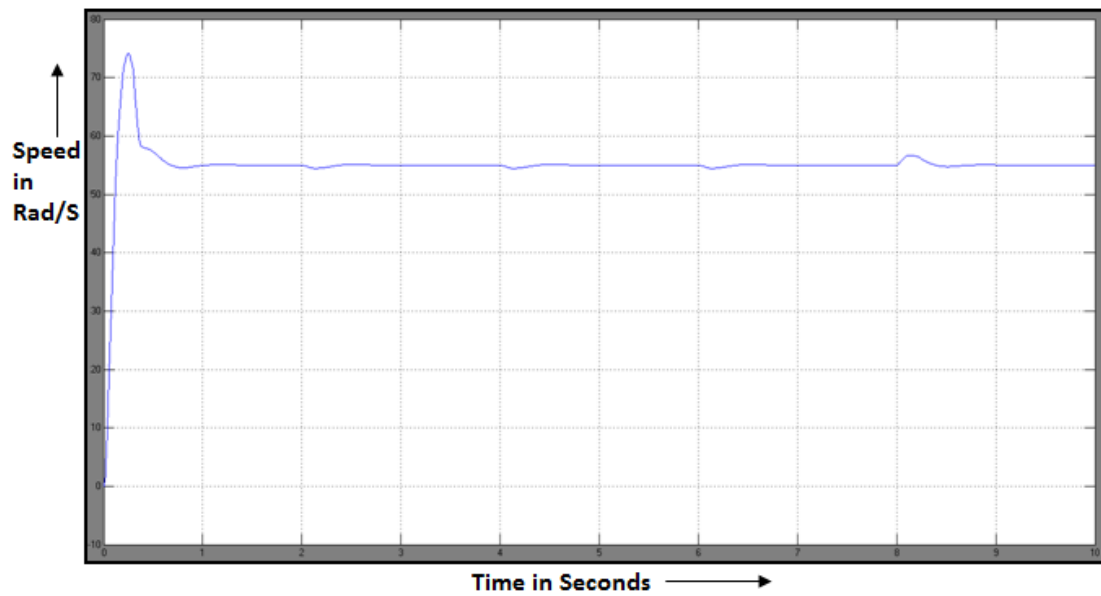
Graph16.Speed Response at stair case type reference speed and constant torque Load:



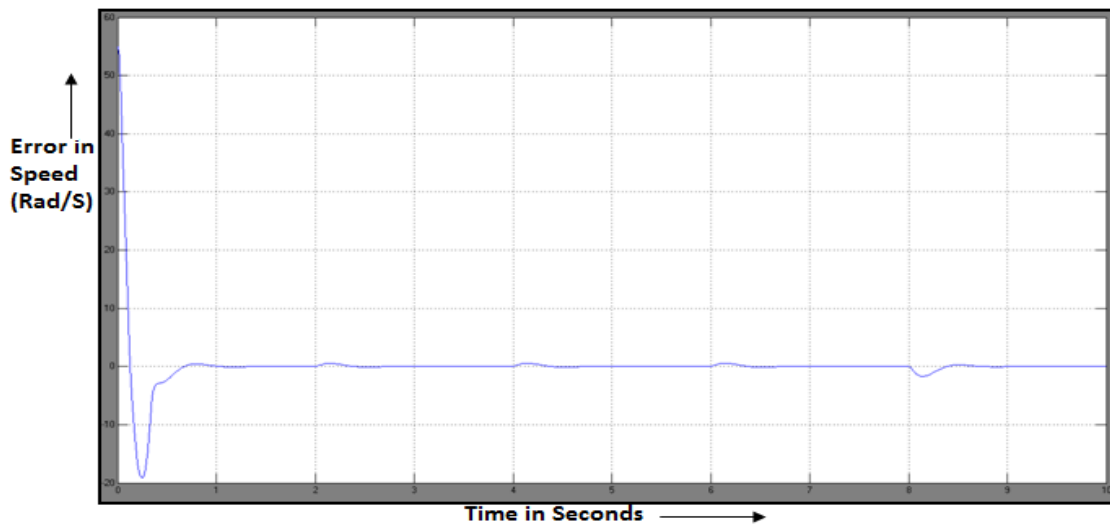
Graph17. Error in Speed Response at stair case type reference speed and constant torque Load:



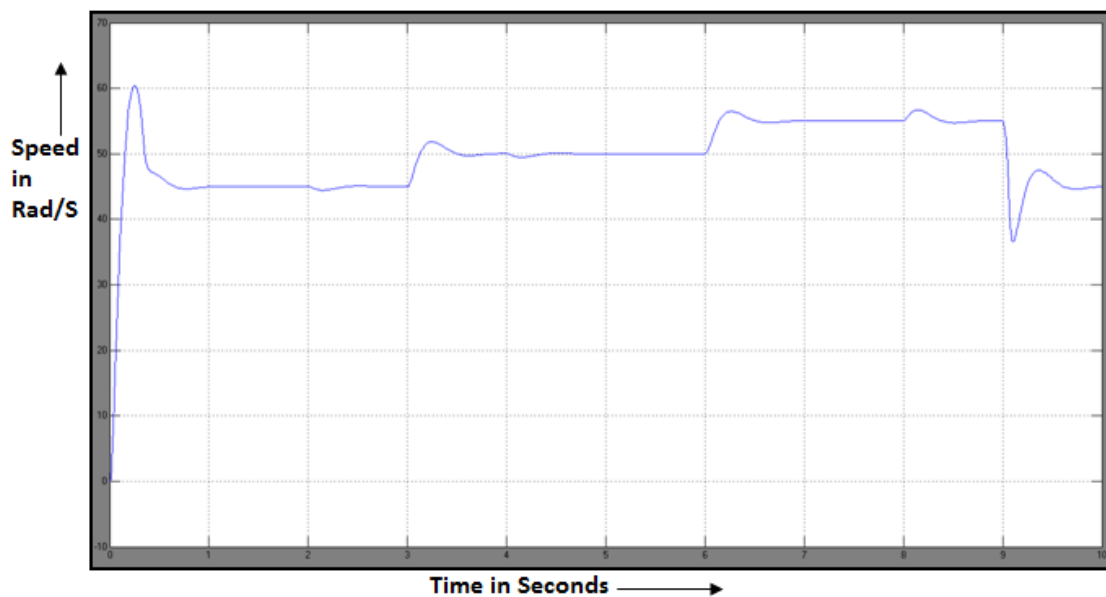
Graph18. Speed Response at reference speed same as rated speed and stair case type torque Load:



Graph19.Error in Speed Response at reference speed same as rated speed and stair case type torque Load:



Graph20.Speed Response at stair case type reference speed and stair case type torque Load:



7.5. Analysis for Graph 6 to 20: When the load is constant the speed response is smooth after attaining steady state. But when the load is varying, speed response have ripples due to time delay in achieving desired speed. When load is constant and reference speed is varying then speed response is shifting accordingly with a time delay. When Reference speed and load is varying then in speed response, speed is adjusting as well as there is some ripple due to delay in achieving current reference speed.

Chapter 8

CONCLUSION

8.1. DISCUSSIONS:

The speed of a dc motor has been successfully controlled by using Chopper as a converter and Proportional-Integral type Speed and Current controller based on closed loop system model. Initially a simplified closed loop model for speed control of DC motor is considered and requirement of current controller is studied. Then a generalized modeling of dc motor is done. After that a complete layout of DC drive system is obtained. Then designing of current and speed controller is done. The optimization of speed control loop is achieved through Modulus Hugging approach. A DC motor specification is taken and corresponding parameters are found out from derived design approach. Ultimately simulation is done for model with and without filter used after reference speed and a comparative study is done on response of both cases. The simulation results under varying reference speed and varying load are also studied and analyzed. The model shows good results under all conditions employed during simulation.

8.2. FUTURE SCOPE:

MATLAB simulation for speed control of separately excited DC motor has been done which can be implemented in hardware to observe actual feasibility of the approach applied in this thesis. This technique can be extended to other types of motors. In this thesis, we have done speed control for rated and below rated speed. So the control for above the rated speed can be achieved by controlling field flux. The problem of overshoot can be removed using a Neural Network and Fuzzy approach.

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